FAST CALCULATION OF A VOLTAGE STABILITY INDEX OF POWER SYSTEMS

Young-Huei Hong
System Planning Department
Taiwan Power Company
Taipei, 100, Taiwan, R. O. C.

Ching-Tsai Pan
Department of Electrical Eng.
National Tsing Hua University
Hsinchu, 300, Taiwan, R. O. C.

Wen-Wei Lin
Institute of Applied Math.
National Tsing Hua University
Hsinchu, 300, Taiwan, R. O. C.

ABSTRACT—The minimum singular value of the Jacobian matrix of the load flow equation may have been preferred as an indicator of voltage collapse when the static voltage stability of power systems is studied. In this paper we propose a highly efficient algorithm to calculate the smallest singular value of a Jacobian matrix of the load flow equation by employing the non-iterative characteristic of an Incremental Condition Estimation (ICE) method and the sparsity characteristic of large scale power networks. Both theoretical basis and computation cost of the algorithm are also detailed in the context. Finally, a practical application example is also given for demonstration.

Keyword: Voltage stability, Stability index, Minimum singular value.

INTRODUCTION

After many voltage collapse events [1-4] of power systems encountered in certain countries, to investigate the voltage instability phenomena has become an important area of research. However, due to the implicit phenomena of the voltage instability, there is no physical quantity available directly by an operator to judge whether the system is secure enough from voltage collapse. Therefore, finding a voltage collapse index has become an important task for many voltage stability studies. Although there are many static indices proposed in the existing literature e.g., [5-9], the minimum singular value of the power flow Jacobian matrix proposed by Thomas and Tiravanich [5] is perhaps one preferred index. However, the applied singular value decomposition method still costs too much computation. Hence, Løf et al. [9] proposed an improved version of the inverse iteration method to fully utilize the sparsity of the Jacobian matrix of power flow equations. Also, if a close initial guess is available then this method can provide a very accurate solution with only few iterations.

Recently, Bischof [10] has proposed an Incremental Condition Estimation (ICE) method for estimating the condition number of a triangular matrix. This method is highly efficient due to its non-iterative characteristic in providing an approximate condition number. In fact, it can also be used directly to obtain an approximate minimum singular value. However, the ICE method can not work when a zero off-diagonal row of the lower triangular matrix is encountered as is often met in power system networks.

In this paper, the authors, based on the non-iterative characteristic of ICE method, first propose an improved method by further exploiting the more efficient block matrix operations to provide a good initial estimate of the minimum singular value, and meanwhile to overcome the disadvantage of ICE method. Then, with only one iteration of Løf’s method, a more accurate minimum singular value can be obtained easily. It is found that the proposed method is rather efficient for applying to finding the desired voltage collapse index.

The rest of the paper can be outlined as follows. In section 2, the theoretical basis of the proposed method is first presented. Then, both computational costs of the proposed method and Løf’s method are given for comparison in section 3. Section 4 gives a practical example to demonstrate the effectiveness of the proposed method. Finally, some conclusions are offered in the last section.

II. THEORETICAL BASIS

In voltage stability analysis, as discussed previously, the singularity of the load flow Jacobian matrix is often adopted as an indicator [5-9]. Theoretically all singular values of a matrix can be derived by utilizing the Singular Value Decomposition (SVD) [11]. That is, let \( J \) be an \( n \times n \) real matrix, then the corresponding singular value decomposition is given as

\[
J = U \Sigma V^T = \sum_{i=1}^{n} \sigma_i u_i v_i^T,
\]

in which \( U \) and \( V \) are \( n \times n \) orthonomal matrices, \( u_i \) and \( v_i \) are the columns of \( U \) and \( V \), respectively, and \( \Sigma = \text{diag}\{\sigma_i, \sigma_i \geq 0, i = 1,2,...,n\} \) where the singular values are ordered so that \( \sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_n = \sigma_{\text{min}} \geq 0 \).

Since the minimum singular value of a matrix can be interpreted as the 2-norm distance of the matrix to the set of all rank-deficient matrices, in voltage stability studies one would often concentrate only on the estimation of its minimum singular value \( \sigma_n \). On the other hand, in power system load flow calculations one usually applies the \( LU \) factorization of the Jacobian matrix \( J \) to find a load flow
solution, where \( L \) is its lower triangular matrix and \( U \) is its upper triangular matrix with unit diagonal. That is, 

\[
J = LU .
\]

By multiplying \( L^{-1} \) and \( J^{-1} \) to both side of the above equation one has \( L^{-1}J^{-1} = L^{-1}LU^{-1}J^{-1} \), or \( L^{-1} = U^{-1}J^{-1} \). This would easily lead to the following derivations:

\[
\sigma_{\min}(L) = \frac{1}{\|L^{-1}\|_2} = \frac{1}{\|U^{-1}\|_2} \geq \frac{1}{\|U\|_2} \geq \sigma_{\max}(U) .
\]

From (3), one has

\[
\sigma_{\min}(J) \leq \sigma_{\min}(L) \cdot \sigma_{\max}(U) .
\]

Obviously, the value of \( \sigma_{\min}(L) \cdot \sigma_{\max}(U) \) in (4) represents the upper bound of \( \sigma_{\min}(J) \), and a minimized value of which gives a good estimation of \( \sigma_{\min}(J) \). In fact, the upper triangular matrix of power flow equations is well conditioned that leads to the value of \( \sigma_{\min}(U) \) relatively large compared with the value of \( \sigma_{\min}(L) \). Therefore, for a matrix near singular, it is quite reasonable to adopt the minimum singular value of \( L \) i.e., \( \sigma_{\min}(L) \) as a good estimation of \( \sigma_{\min}(J) \). The problem of finding a minimum single value of a matrix \( J \) is thus switched to finding the minimum singular value of the corresponding matrix \( L \). As to estimating the minimum singular value of a triangular matrix more efficiently, an enhanced algorithm is proposed as follows.

In order to estimate \( \hat{\sigma}_{\min}(L) \) efficiently, one begins with an initial sub-matrix \( I \) of arbitrary dimension \( m \) from the left upper part of the matrix, and then process the estimation of the sub-matrix with \( k \) more rows as shown in (5), where \( \hat{\sigma}_{\min}(I) \) is supposed to have been estimated beforehand. That is, one starts with estimating \( \sigma_{\min}(I) \) first, and then estimates the minimum singular value of the sub-matrix \( l_{src} \) (formed by augmenting \( k \) rows to \( l \) matrix) by application of some previous results. This process is continued until the complete \( L \) matrix is done.

To be more clearly, one may represent \( L \) matrix as shown in the following:

\[
L_{\text{src}} = \begin{pmatrix}
I & 0 & \ldots & 0 \\
W^T & \Gamma & \ldots & \Gamma \\
\vdots & \vdots & \ddots & \vdots \\
W^T & \Gamma & \ldots & \Gamma \\
\end{pmatrix}
\]

in which the augmented sub-matrix \( l_{src} \) is defined as

\[
l_{src} = \begin{pmatrix}
I & 0 \\
W^T & \Gamma \\
\end{pmatrix}
\]

where the augmented matrix \( l_{src} \in \mathbb{R}^{(m+k) \times (m+k)} \) and \( \Gamma \in \mathbb{R}^{k \times k} \) are lower triangular matrices and \( W \in \mathbb{R}^{m \times k} \). Given a unit column vector \( x \) of dimension \( m \), and let \( d = \alpha x \) also of dimension \( m \), then \( \sigma_{\min}(I) \leq \alpha \|x\|_2 \). Thus, if one can find a vector \( d \) with minimum norm, then \( \sigma_{\min}(I) = \|d\|_2 \).

Knowing the above relation between \( \sigma_{\min}(L) \) and minimum \( \|d\|_2 \), one can find a new vector \( d_{src} \) with dimension \((m+k)\times 1\) and with minimum norm from the following equation

\[
d_{src} = l_{src}x_{src} = \begin{pmatrix}
I & 0 \\
W^T & \Gamma \\
\end{pmatrix}
\begin{pmatrix}
x_0 \\
C
\end{pmatrix}
\]

where \( s \) is a scalar and \( C \) is a vector of dimension \( k \times 1 \) such that \( x_{src} \) is with unity norm, i.e.,

\[
s^2 + C^TC = 1
\]

Equivalently, minimum norm of \( d_{src} \) can be obtained by solving the following minimization problem with objective function

\[
\|d_{src}\|_2^2 = \begin{pmatrix} s & C^T \end{pmatrix} \begin{pmatrix} S \\ C \end{pmatrix}
\]

subject to the equality constraint (8). The resulting solution can be derived by using Largrange multiplier method. Note that the \( S \) matrix in equation (9) is a symmetrical and semi-definite matrix. It turns out that the unknown vector of \( \begin{pmatrix} s & C^T \end{pmatrix} \) is just the unity eigenvector corresponding to the minimum eigenvalue of \( S \), and hence from equation (9) one can obtain

\[
\sigma_{\min}(l_{src}) = \|d_{src}\|_2 = \sqrt{\lambda_{\min}(S)}
\]

and

\[
x_{src} = \begin{pmatrix}
x_0 \\
C
\end{pmatrix}
\]

Thus, given \( x \), \( \|d\|_2 \), \( W^T \) and \( \Gamma \), one can find the minimum eigenvalue of \( S \) with much smaller dimension and the corresponding unity eigenvector easily. Then, \( \sigma_{\min}(l_{src}) \) and \( x_{src} \) can be calculated directly. This process is repeated until the \( L \) matrix is done and therefore \( \hat{\sigma}_{\min}(L) \) is obtained. For higher order \( S \) matrix, the power iteration method[11] can be used to find the desired eigenvalue and the eigenvector efficiently. In practice, it is seen that although the above mentioned algorithm is efficient and accurate enough in estimating the exact minimum singular value of matrix \( L \) near singular, however, when \( L \) is not close to singularity, more accurate estimate of \( \sigma_{\min}(L) \) may not be achieved. As a result, it is suggested that a step correction of the solution be proceeded. To implement, one may apply the eigenvector \( x \) given in (12) as an initial vector of the \( L \) \( \hat{\sigma} \)'s method[9], a more accurate minimum singular value of \( J \) can thus be achieved.

III. ALGORITHM AND COMPUTATION FLOPS

As discussed previously, the proposed method consists of two steps. First, a nice estimate of the minimum singular value of \( L \) is obtained very efficiently. Then, the final \( x_{src} \) vector of the first step is used as an initial vector of \( L \) \( \hat{\sigma} \)'s method to calculate a more accurate minimum singular value of \( J \).
value of $J$. More precisely, let $n$ be the dimension of $L$ and $k$ be the chosen block size, such that

$$ n-1 = kq + r, \quad (13) $$

where $q$ and $r$ are positive integers. In case $r = 0$, the proposed method may be initialized by choosing $x^{(0)} = 1$ and $\|d^{(0)}\|_2 = 1$. Then the $S^{(0)}$ matrix may be formed, and the minimum eigenvalue and the corresponding eigenvector are calculated. This process, called $k$-block process, is repeated until $L$ is completed and one more step of $\hat{L}\hat{O}f$’s method is executed for $J$ matrix.

On the other hand, if $r = 0$, then after the initialization process and before the repeated $k$-block process is executed, a $r$-block process has to be executed. The remaining part then follows the same process as that of $r = 0$ case. Thus, for a given matrix $J$ and its factorized lower triangular matrix $L \in R^{n \times n}$, the proposed algorithm can therefore be described as follows:

1. Input $n, k, q, r$, and let $j = 0$, $x^{(0)} = 1$, $\|d^{(0)}\|_2 = 1$.

2. If $r = 0$, then go to step 5.

   Otherwise form

   $$ S_\theta = \begin{bmatrix} x^{(0)}W_0T_0 & x^{(0)}W_0^T & x^{(0)}T_0 \\ T_0 & W_0T_0 & T_0 \\ \Gamma_0 & \Gamma_0 & \Gamma_0 \end{bmatrix} \in R^{(r+1)(r+1)} \quad (14) $$

   where

   $$ W_0 = \begin{bmatrix} I_{11} \\ \vdots \\ I_{r+1,1} \end{bmatrix} \in R^{r+1}, \text{and } \Gamma_0 = \begin{bmatrix} I_{22} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ I_{r+1,2} & \cdots & I_{r+1,r+1} \end{bmatrix} \in R^{r+1 \times r} \quad (15) $$

3. Find $\lambda_{\text{min}}(S_\theta)$ and the unity eigenvector

   $$ \begin{bmatrix} s_0 & C_0 \end{bmatrix} \in R^{(r+1) \times 1}. $$

4. Update $\|d^{(0)}\|_2 \leftarrow \lambda_{\text{min}}(S_\theta)$ and $x^{(0)} \leftarrow \begin{bmatrix} s_0 \end{bmatrix} x^{(0)}$.

5. Form

   $$ S^{(j)} = \begin{bmatrix} x^{(j)} & (W^{(j-i)}(W^{(j-i)})^T)x^{(j)} & (W^{(j-i)}(W^{(j-i)})^T)x^{(j)} & (W^{(j-i)}(W^{(j-i)})^T)x^{(j)} \\ (T^{(j-i)})^T & (T^{(j-i)})^T & (T^{(j-i)})^T & (T^{(j-i)})^T \end{bmatrix} \in R^{(r+1)(r+1)} \quad (16) $$

   where

   $$ (W^{(j-i)})^T = \begin{bmatrix} I_{r+i,j}, & \cdots, & I_{r+i,j+k-1} \\ I_{r+i+1,j}, & \cdots, & I_{r+i+1,j+k-1} \end{bmatrix} \quad (17) $$

   and

   $$ \Gamma^{(j-i)} = \begin{bmatrix} I_{r+j,k+2}, & \cdots, & I_{r+j,k+r+1} \\ I_{r+j+1,k+2}, & \cdots, & I_{r+j+1,k+r+1} \end{bmatrix}. \quad (18) $$

6. Find $\lambda_{\text{min}}(S^{(j-1)})$ and the corresponding unity eigenvector

   $$ \begin{bmatrix} s^{(j-1)} \end{bmatrix} \in R^{(r+1) \times 1}. $$

7. Update $x^{(j-1)} \leftarrow \begin{bmatrix} s^{(j-1)}x^{(j)} \\ C^{(j-1)} \end{bmatrix}$ and $\|d^{(j)}\|_2 \leftarrow \lambda_{\text{min}}(S^{(j-1)})$.

8. If $j = q$, then $\hat{\sigma}_{\text{min}}(L) = \sqrt{\lambda_{\text{min}}(S^{(q)})}$, go to step 10.

9. Let $j = j + 1$, go to step 5.

10. Solve $U^Ty = x^{(j)}$ for $y$.

11. Solve $U^Tw = x^{(j)}$ for $u$, and $\sigma_{\text{min}}(J) = \|u\|_2$.

12. Stop.

Note that at step 3 or 6 in the above algorithm, one may apply power iteration method [11] to find the minimal eigenvalue and eigenvector of $S$. However, the computation can be reduced if $S$ is pre-permuted as

$$ S = \begin{bmatrix} P & G^T & P^T \end{bmatrix}, $$

where $P$ and $G$ are defined as follows:

$$ P = \begin{bmatrix} 0 & 1 \\ I_n & 0 \end{bmatrix}, $$

$$ G = \begin{bmatrix} (\Gamma)^T \\ \sqrt{d'x}W \end{bmatrix}. \quad (21) $$

Thus, instead of calculating the minimum eigenvalue of $S$ directly, it is more efficient to find the minimum eigenvalue of $G$.

To demonstrate the efficiency of the proposed algorithm, the computation costs for the algorithm and the $\hat{L}\hat{O}f$’s method are shown in Table 3.1 and Table 3.2, respectively. At the first column of both tables, the main operations of each algorithm are listed. At the second column, the total operation counts of each associated operator are given. Here, one flop means one multiplication and one addition. However since an addition spends much less CPU time in comparing with a multiplication, the computation flops associated with the addition operators are ignored when one compares the computation costs of these algorithms. Moreover, the operation cost of a square root is reasonably approximated in this paper as 10 flops for simplicity. From practical experience, $\hat{L}\hat{O}f$’s method needs 5 iterations on average to converge to the solution with an error less than 1.0e-05 by using a 486 PC with double precision. Therefore, the total computation cost of the $\hat{L}\hat{O}f$’s method as shown in Table 3-1 is calculated as per $5(6n^2 + 8n + 18) = 30n^2 + 40n + 90$ flops.

In Table 3.2, for a matrix of arbitrary dimension $n$, and a chosen block size $k$, the flops for $r = 0$ and $r \neq 0$ by using the proposed algorithm are shown respectively. In the second row of the Table, it is assumed that 5 iterations of power method [11] are used for getting the minimum eigenvalue and corresponding eigenvector of matrix $G$. Totally, for the case of $r \neq 0$, the computation counts $T_r(n,k)$ is

$$ T_r(n,k) = \frac{q + 29}{2} k^2q + \frac{q + r + 101}{2} kq + q(r + 81) + (15r^2 + 51r + 81). \quad (22) $$
In case \( r = 0 \), then the computation counts \( T_2(n, k) \) becomes

\[
T_2(n, k) = \frac{q + 29}{2} k^2 q + \frac{q + 101}{2} k q + 81 q .
\] (23)

**Table 3-1 The computation counts of the \( L \hat{O} f \)'s method**

<table>
<thead>
<tr>
<th>Computation</th>
<th>Flops</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V^{(x)}_{x} )</td>
<td>( 3n + 9 )</td>
<td>For each element of the vector there are ( n ) additions, ( n - 1 ) multiplications and 1 square-root operation.</td>
</tr>
<tr>
<td>( U^T z = y )</td>
<td>( \frac{n(3n + 1)}{2} )</td>
<td>Backward substitution ( \sum_{i=n}^{1} (3n - 1) ) flops</td>
</tr>
<tr>
<td>( L^T y = \hat{w} )</td>
<td>( \frac{n(3n + 1)}{2} )</td>
<td>Forward substitution ( \sum_{i=1}^{n} (3n - 1) ) flops</td>
</tr>
<tr>
<td>( \lambda_{mi}^{s}(G) )</td>
<td>( \frac{5q(3k^2 + 16k + 16)}{2} )</td>
<td>Calculate the min. eigenvalue and eigenvector of matrix ( G ) with dimension ( k + 1 ).</td>
</tr>
<tr>
<td>( x^{(i+1)} = \frac{ax^{(i)}}{c} + \frac{bq}{2} )</td>
<td>( \frac{q(r + 1) + \frac{kq(q - 1)}{2}}{c} )</td>
<td>Update vector ( x ).</td>
</tr>
<tr>
<td>( Lw = \hat{w} )</td>
<td>( \frac{n(3n + 1)}{2} )</td>
<td>Forward substitution ( \sum_{i=1}^{n} (3n - 1) ) flops</td>
</tr>
<tr>
<td>( \sigma_{\min}(L) )</td>
<td>( 6n^2 + 8n + 18 )</td>
<td>One iteration of ( L \hat{O} f )'s method</td>
</tr>
<tr>
<td>Total</td>
<td>( 30n^2 + 40n + 90 )</td>
<td>Approximated with 5 iterations</td>
</tr>
</tbody>
</table>

From equation (22) and (23) one can see that the block size \( k \) affects the efficiency of the algorithm at the stage of calculating \( \sigma_{\min}(L) \). In order to demonstrate the efficiency for different block sizes, the total flops count \( T(n, k) \) of the proposed algorithm of (22) is plotted in Fig. 3-1. It is observed that for any specific \( n \) one may choose a proper \( k \) to have the least computations. On the other hand, if one correction step of \( L \hat{O} f \)'s method is appended, the total flops will be \( T(n, k) + 6n^2 + 8n + 18 \), where the \( T(n, k) \) part is very negligible. Even so, compared with the \( L \hat{O} f \)'s method, it is seen that the proposed method is still less expensive. In fact, for larger systems the computation cost reduction of the proposed method gets even larger. Since the encountered power systems are always large, therefore, the proposed algorithm is very attractive for applications in power system voltage stability index calculations. Since \( L \hat{O} f \)'s method is already very efficient, the computational time of \( \sigma_{\min} \) is much less than that of a load flow solution, for clarity, instead of including the load flow solution time, the total computation flops of the proposed algorithm and the \( L \hat{O} f \)'s method in terms of the dimension is given in Fig. 3-2 for comparison. For the example system consisting of 305 buses, 506 lines given in the next section, the computation time of the proposed method is about 1/30 of one load flow solution time.

**Table 3-2 The computation counts of the proposed method**

<table>
<thead>
<tr>
<th>Computation</th>
<th>Flops</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w \cdot x )</td>
<td>( \frac{1}{2} k^2 q(q - 1) + kq(r + 1) )</td>
<td>Calculate elements of matrix ( G ).</td>
</tr>
<tr>
<td>( \lambda_{mi}^{s}(G) )</td>
<td>( 5q(3k^2 + 10k + 16) )</td>
<td>Calculate the min. eigenvalue and eigenvector of matrix ( G ) with dimension ( k + 1 ).</td>
</tr>
<tr>
<td>( x^{(i+1)} = \frac{ax^{(i)}}{c} + \frac{bq}{2} )</td>
<td>( \frac{q(r + 1) + \frac{kq(q - 1)}{2}}{c} )</td>
<td>Update vector ( x ).</td>
</tr>
<tr>
<td>( T(n, k) = A + B + C + D )</td>
<td>( 6n^2 + 8n + 18 )</td>
<td>One iteration of ( L \hat{O} f )'s method</td>
</tr>
<tr>
<td>Total</td>
<td>( T(n, k) + D )</td>
<td>Approximated with 5 iterations</td>
</tr>
</tbody>
</table>

![Fig. 3-1 The flops count \( T(n,k) \) of the proposed algorithm at the blocked stage with respect to the block size and matrix dimension.](image1)

![Fig. 3-2 The comparison of the computation flops of the proposed algorithm and the \( L \hat{O} f \)'s method.](image2)
IV. NUMERICAL EXAMPLE
The proposed algorithm has been implemented successfully in the System Planning Department of Taiwan Power Company on a personal computer using FORTRAN language. The first application involves the daily voltage stability analysis of Taipower system during summer peak load. Where the proposed algorithm is used to evaluate the minimum singular value trajectory of a particular load growth pattern. Due to the excellent efficiency of the algorithm, the computation time can be neglected compared with the load flow calculations. The second application involves the evaluation of the relative effectiveness of voltage stability improvement or deterioration due to line outage or addition of a new transmission trunk, and addition of capacitor banks. The percentage change of the minimum singular value is used for evaluation.

As an illustration, some voltage stability analyses of the Taipower system in 1995 are given below as an example. This system has an installed capacity of 21,898 Mw and the peak load demand at summer is 19,933 Mw. It is seen that less than 10% of spinning reserve is available. The load center of this longitudinal island is located at the northern part. More than 3000Mw power is transported from the central and the southern Taiwan through 4 circuits, as can be seen in Fig 4.1. The system includes 305 buses, 506 lines, 62 generators and 151 transformers.

![Diagram of EHV trunk system of TPC](image)

First consider the case of studying the influence on voltage stability due to outage of an important transmission line or addition of an extra paralleling circuit with equal capacity. Due to limitation of space Table 4-1 only shows the percentage change of $\sigma_{min}$ for some important transmission lines under consideration. The normal line flows in these lines during the peak summer load are also shown on the same table for reference. From Table 4-1, one can see that if line #11 is tripped, $\Delta\sigma_{min}$ will be most significant meaning outage of this line will result in most impact on voltage stability deterioration which results in the system margin of 891 Mw. (The margin for the case without tripping lines is 1269Mw.) On the other hand, to improve the voltage stability from planning view point, addition of a parallel circuit with line #11 will also result in most significant improvement, namely 1543 Mw margin from collapse.

<table>
<thead>
<tr>
<th>Lines</th>
<th>Base Case Flows</th>
<th>Rated Mw</th>
<th>Trip.</th>
<th>Add.</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>Q</td>
<td>$\Delta\sigma_{min}$ (%)</td>
<td>Margin Mw</td>
<td>$\Delta\sigma_{min}$ (%)</td>
</tr>
<tr>
<td>8</td>
<td>354 120</td>
<td>2140</td>
<td>-0.089</td>
<td>1180</td>
</tr>
<tr>
<td>9</td>
<td>88 57</td>
<td>1196</td>
<td>-0.518</td>
<td>1003</td>
</tr>
<tr>
<td>10</td>
<td>584 26</td>
<td>2140</td>
<td>-1.463</td>
<td>926</td>
</tr>
<tr>
<td>11</td>
<td>480 17</td>
<td>1196</td>
<td>-1.987</td>
<td>891</td>
</tr>
<tr>
<td>19</td>
<td>282 18</td>
<td>1196</td>
<td>-0.456</td>
<td>992</td>
</tr>
<tr>
<td>20</td>
<td>36 3</td>
<td>1196</td>
<td>-0.145</td>
<td>1172</td>
</tr>
<tr>
<td>21</td>
<td>450 36</td>
<td>2140</td>
<td>-0.322</td>
<td>1034</td>
</tr>
<tr>
<td>23</td>
<td>212 11</td>
<td>1196</td>
<td>-0.098</td>
<td>1219</td>
</tr>
</tbody>
</table>

Second consider the case of studying the effect of adding capacitor banks at 69 KV buses. Similarly the percentage change of the resulting minimum singular value is used as a relative voltage stability index. The same system data at summer peak load is employed. The results are shown in Table 4-2 for two load levels, namely 18,300 and 19100 Mw respectively. It is seen from Table 4-2 that addition of a capacitor bank at WUFEN substation of Center Taiwan will result in the greatest improvement in voltage stability in both load levels. Also, the percentage changes of the minimum singular value of the 19100Mw load level are higher than the first one reflects the necessity of the installing additional capacitors.

Finally, it may be worth mentioning that while studying the voltage stability index profile for load growth and when generators are hitting reactive power limits the dimension of the load flow Jacobian matrix is change in the same way as mentioned in [9].
Table 4-2 The % change of the minimum singular value of primary substations/161/69kV) for installing capacitor of 4Mvar.

<table>
<thead>
<tr>
<th>Loading conditions</th>
<th>18300Mw Load level</th>
<th>19100Mw Load level</th>
</tr>
</thead>
<tbody>
<tr>
<td>area</td>
<td>Station</td>
<td>( \Delta \sigma_{\text{min}} (%) )</td>
</tr>
<tr>
<td>North</td>
<td>TINGHOU</td>
<td>0.0708</td>
</tr>
<tr>
<td></td>
<td>NANGANG</td>
<td>0.0565</td>
</tr>
<tr>
<td></td>
<td>TAINAN</td>
<td>0.0775</td>
</tr>
<tr>
<td></td>
<td>TAIPEI</td>
<td>0.1124</td>
</tr>
<tr>
<td></td>
<td>HSINCHEN</td>
<td>0.1120</td>
</tr>
<tr>
<td></td>
<td>WULIN</td>
<td>0.1855</td>
</tr>
<tr>
<td></td>
<td>NANTOU</td>
<td>0.0679</td>
</tr>
<tr>
<td></td>
<td>NANCHANG</td>
<td>0.1368</td>
</tr>
<tr>
<td></td>
<td>CHICHIA</td>
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V. CONCLUSIONS

In this paper, based on the non-iterative characteristic of ICE method and employing the sparsity characteristic of power systems, the authors propose a highly efficient method for finding an accurate estimate of minimum singular value of the load flow Jacobian matrix as a voltage stability index. The proposed method can also overcome the difficulty of ICE method when a lower triangular matrix contains a zero off-diagonal row. This algorithm has also been implemented successfully on a personal computer and is rather useful for voltage stability evaluation in system planning and operation. Both theoretical basis and computational cost analysis are given in the context. Finally, a practical example is given for illustration. It is seen that the proposed method is rather suitable for applying to large scale power system voltage stability analysis if the minimum singular value is adopted as a voltage stability index.

REFERENCES


Young-Huei Hong was born in Tainan, Taiwan, 1956. He received his BSEE degree in 1980 from Tsing Hua University, Hsinchu, Taiwan, and MSEE degree in 1985 from Cheng Kung University, Tainan, Taiwan. After that, he served in System Planning Department, Taiwan Power Company. At present, he is a Ph.D student in Institute of E. E., Tsing Hua University. His main research interests are the power system analysis, planning and power quality.

Ching-Tsai Pan was born in Taipei, Taiwan, Republic of China, in October 1948. He received the B.S. degree in Electrical Engineering from the National Cheng Kung University, Tainan, Taiwan, in 1970, and the M.S. and Ph.D degrees from Texas Tech University, Lubbock, Texas, in 1974 and 1976, respectively, all in Electrical Engineering. Since 1977, he has been with the Department of Electrical Engineering, National Tsing Hua University, Hsinchu, Taiwan, where he is currently Professor. From 1985 to 1986, he was a Visiting Professor at the Department of Electrical Engineering, Ecole Centrale de Lyon in France. His research interests are in the areas of power systems, control systems, numerical analysis and power electronics. Dr. Pan is member of IEEE, ICE, IEE, Phi Tau Phi, Eta Kappa Nu, and Phi Kappa Phi.

Wen-Wei Lin was born in Taipei, Taiwan, 1953. He received his BS degree in 1976 from Chengchi University, Taipei, MS degree in 1978 from Tsinghua University, Hsinchu, Taiwan, and Ph.D degree in 1985 from Bielefeld University, Germany. At present he is a Professor in Institute of Applied Mathematics of Tsing Hua University, Hsinchu, Taiwan.